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VARIATIONAL PRINCIPLE IN THE PROBLEM OF SHIP PROPULSION AND POWER PLANT OPERATION WITH RESPECT TO SUBJECTIVE PREFERENCES

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It is elaborated a theoretical background for the simplest case of a variational problem with the consideration of a ship's propulsion and her power plant operators' individual subjective preferences and their entropy; operational function and its derivative with respect to time. In the given case, it is obtained the generalized expressions of the functional; canonical distributions of subjective preferences; transformed Euler-Lagrange equation; and differential equation of the second order.
Keywords: subjective analysis, ship propulsion operation, power plant, subjective entropy, individual preferences, multi-alternative situations, Euler-Lagrange equation, canonical distributions, variational principle.

Introduction. Multi-alternativeness of operational situations imposes making managing decisions in uncertainty. Sometimes it drastically changes modes of a ship's propulsion and her power plant operation. In some cases wrong decisions may lead to tragedies.

Urgency of researches. The urgency of researches in the field of human factor influence upon safety is well known, the same to researches of optimal economical modes of operation. In science, it is always an important problem to find a certain function that delivers an extremum to the considered functional.

Thus, the problem of finding such a function, in conditions that consider uncertainty of marine ships' power plants and their propulsions of transport vessels operators' subjective preferences in multi-alternative operational situations, is a complex and actual one.

Analysis of the latest researches and publications. Since the previous fundamental works of [1, 2] publication, the concept of subjective preferences have been successfully applied to solving a lot of problems in which there were considerations of such or another type of uncertainty. Some of the latest researches and publications are [3-5]. Where it was suggested the hybrid pseudo-entropy function; complex criterion of a voyage profit expectation, involving the complete probability of achieving the corresponding alternative with respect to its individual's subjective preference; a model of the number of seafarers on board and ashore estimation accordingly to the competitors' subjective preferences.

The theory of subjective probabilities and the best optimal decisions was elaborated not so long ago in the publication [6]. There was not a notion of an individual subjective preference there. It appeared in the works [1, 2] as a result of researches that could not be conducted with the subjective probabilities of [6]. In the works [1, 2] the distributions functions of the individual subjective preferences were obtained mathematically on the basis of the ideas of the earlier publications [7-9] of the predecessors.

The task setting. Thus, the purpose of this paper is to theoretically generalize and expand the application of the variational principle to solving the problems which make allowance for operators' individual subjective preferences and their entropy, and that, in its turn, allows obtaining the needed differential equations of the second order for the operational functions in the dynamics of the situations developments.

The main content (material). On building the simplest functional which includes a distribution of the operators' (active elements' of a system/subjects'/decision making persons') subjective preferences, subjective entropy of the individual preferences, operational function, its derivative with respect to time, and normalizing condition; we solve a variational problem obtaining the generalized result as a differential equation of the second order.

The problem formulation. The postulated functional is [1, P. 119, (3.38)]

$$\Phi_\pi = \int_{t_0}^{t_1} \left(-\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left[\sum_{i=1}^{N} \pi_i(t) - 1 \right] \right) dt \qquad (1)$$

where t – time; $-\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t)$ – entropy of subjective preferences of $\pi_i(t)$; N – number of the achievable alternatives; β, γ – structural parameters (can be considered in different situations as Lagrange coefficients or weight coefficients), they reflect the endogenous parameters of psych; F_i – subjective efficiency function of an i[th] alternative; $\sum_{i=1}^{N} \pi_i(t) - 1$ – normalizing condition.

In the simplest problem setting we consider $x(t)$ and $\dot{x}(t)$ as the subjective efficiency functions of the two achievable alternatives with the corresponding preferences of $\pi_1(t)$, $\pi_2(t)$. With respect to particular combinations of $x(t)$, $\dot{x}(t)$, $x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$, we will get the eleven specific variants of the functional (1), which have their common general view of:

$$\Phi_\pi = \int_{t_0}^{t_1} \left(-\sum_{i=1}^{N=4} \pi_i(t) \ln \pi_i(t) + \beta \left[\pi_1(t) x(t) + \alpha_2 \pi_2(t) \dot{x}(t) + \alpha_3 \pi_3(t) x(t) \dot{x}(t) + \right. \right.$$
$$\left. \left. + \alpha_4 \pi_4(t) \frac{\dot{x}(t)}{x(t)} \right] + \gamma \left[\sum_{i=1}^{N=4} \pi_i(t) - 1 \right] \right) dt, \qquad (2)$$

where α_i – coefficients that consider differences in measurement units. The last functional (2) is the general one and each of the previously mentioned specific variants derives from it with the corresponding $\pi_i(t)$ and α_i.

Namely, we can obtain corresponding cognitive functions with respect to the particular cases of the mentioned above subjective efficiency functions as the operational function, its control, and their possible combinations for the eleven variants in the view of:

$$\beta[\pi_1(t)x(t) + \alpha\pi_2(t)\dot{x}(t)], \; \beta[\pi_1(t)x(t) + \alpha\pi_2(t)x(t)\dot{x}(t)],$$

$$\beta\left[\pi_1(t)x(t) + \alpha\pi_2(t)\frac{\dot{x}(t)}{x(t)}\right], \; \beta[\pi_1(t)\dot{x}(t) + \alpha\pi_2(t)x(t)\dot{x}(t)],$$

$$\beta\left[\pi_1(t)\dot{x}(t) + \alpha\pi_2(t)\frac{\dot{x}(t)}{x(t)}\right], \; \beta\left[\pi_1(t)x(t)\dot{x}(t) + \alpha\pi_2(t)\frac{\dot{x}(t)}{x(t)}\right],$$

$$\beta[\pi_1(t)x(t) + \alpha_2\pi_2(t)\dot{x}(t) + \alpha_3\pi_3(t)x(t)\dot{x}(t)],$$

$$\beta\left[\pi_1(t)x(t) + \alpha_2\pi_2(t)\dot{x}(t) + \alpha_3\pi_3(t)\frac{\dot{x}(t)}{x(t)}\right],$$

$$\beta\left[\pi_1(t)x(t) + \alpha_2\pi_2(t)x(t)\dot{x}(t) + \alpha_3\pi_3(t)\frac{\dot{x}(t)}{x(t)}\right],$$

$$\beta\left[\pi_1(t)\dot{x}(t)+\alpha_2\pi_2(t)x(t)\dot{x}(t)+\alpha_3\pi_3(t)\frac{\dot{x}(t)}{x(t)}\right].$$

Thus, in any interested case for researching, for instance, the speed of rotation, its control (the rate of the speed of rotation change), and certain combinations of their interrelations may embody a given set of operational alternatives. Therefore, using the considered functional in the general view of (2) we represent the needed operational multi-alternative situations, modeling them with the help of the corresponding preferences and measurement units coefficients; the required cognitive functions, as well as the optimized specific operational functionals.

The problem solution. Applying the necessary conditions for extremums in the view of Euler-Lagrange equations

$$\frac{\partial R^*}{\partial \pi_i}-\frac{d}{dt}\left(\frac{\partial R^*}{\partial \dot{\pi}_i}\right)=0,\ \frac{\partial R^*}{\partial x}-\frac{d}{dt}\left(\frac{\partial R^*}{\partial \dot{x}}\right)=0, \tag{3}$$

where R^* – the under-integral function of the corresponding integral (2), we get the corresponding expressions of canonical distributions of the preferences

$$\pi_1=\frac{e^{\beta x}}{e^{\beta x}+e^{\alpha_2\beta\dot{x}}+e^{\alpha_3\beta x\dot{x}}+e^{\alpha_4\beta\frac{\dot{x}}{x}}},\ \pi_2=\frac{e^{\alpha_2\beta\dot{x}}}{e^{\beta x}+e^{\alpha_2\beta\dot{x}}+e^{\alpha_3\beta x\dot{x}}+e^{\alpha_4\beta\frac{\dot{x}}{x}}},$$

$$\pi_3=\frac{e^{\alpha_3\beta x\dot{x}}}{e^{\beta x}+e^{\alpha_2\beta\dot{x}}+e^{\alpha_3\beta x\dot{x}}+e^{\alpha_4\beta\frac{\dot{x}}{x}}},\ \pi_4=\frac{e^{\alpha_4\beta\frac{\dot{x}}{x}}}{e^{\beta x}+e^{\alpha_2\beta\dot{x}}+e^{\alpha_3\beta x\dot{x}}+e^{\alpha_4\beta\frac{\dot{x}}{x}}}. \tag{4}$$

Now, again, the particular cases can be obtained from the general forms of the canonical distributions of (4) if one uses the necessary measurement units coefficients correspondingly to the considered multi-alternativeness of a specific operational situation formed in an active system.

From (2) upon the condition of (3), for all particular cases

$$\pi_1=\alpha\dot{\pi}_2,\ \pi_1=\alpha\dot{\pi}_2 x,\ \pi_1=\alpha\left(\frac{\dot{\pi}_2}{x}\right),\ \dot{\pi}_1=-\alpha\dot{\pi}_2 x,\ \dot{\pi}_1=-\alpha\left(\frac{\dot{\pi}_2}{x}\right),$$

$$\dot{\pi}_1=-\alpha\left(\frac{\dot{\pi}_2}{x^2}\right),\ \pi_1=\alpha_2\dot{\pi}_2+\alpha_3\dot{\pi}_3 x,\ \pi_1=\alpha_2\dot{\pi}_2+\alpha_3\frac{\dot{\pi}_3}{x},\ \pi_1=\alpha_2\dot{\pi}_2 x+\alpha_3\frac{\dot{\pi}_3}{x},$$

$$\dot{\pi}_1=-\alpha_2\dot{\pi}_2 x-\alpha_3\frac{\dot{\pi}_3}{x},\ \pi_1=\alpha_2\dot{\pi}_2+\alpha_3 x\dot{\pi}_3+\frac{\alpha_4}{x}\dot{\pi}_4. \tag{5}$$

For the generalized equation by Euler-Lagrange in the transformed view (the last one in (5)) the generalized differential equation of the second order will be

$$\ddot{x}=\frac{\pi_1+A+B+C}{D+E+F}, \tag{6}$$

where

$$A=\alpha_2\left\{\beta\pi_2\left(\pi_1+\alpha_3\dot{x}\pi_3-\alpha_4\frac{\dot{x}}{x^2}\pi_4\right)\dot{x}\right\},$$

$$B=-\alpha_3 x\left\{\beta\pi_3\left[\alpha_3\dot{x}(\pi_1+\pi_2+\pi_4)-\pi_1+\alpha_4\frac{\dot{x}}{x^2}\pi_4\right]\dot{x}\right\},$$

$$C=-\frac{\alpha_4}{x}\left\{\beta\pi_4\left[-\alpha_4\frac{\dot{x}}{x^2}(\pi_1+\pi_2+\pi_3)-\pi_1-\alpha_3\dot{x}\pi_3\right]\dot{x}\right\},$$

$$D=\alpha_2\left\{\beta\pi_2\left[\alpha_2(\pi_1+\pi_3+\pi_4)-\alpha_3 x\pi_3-\frac{\alpha_4}{x}\pi_4\right]\right\},$$

$$E=\alpha_3 x\left\{\beta\pi_3\left[\alpha_3 x(\pi_1+\pi_2+\pi_4)-\alpha_2\pi_2-\frac{\alpha_4}{x}\pi_4\right]\right\},$$

$$F=\frac{\alpha_4}{x}\left\{\beta\pi_4\left[\frac{\alpha_4}{x}(\pi_1+\pi_2+\pi_3)-\alpha_2\pi_2-\alpha_3 x\pi_3\right]\right\}, \qquad (7)$$

where $\pi_1(t)\ldots\ \pi_4(t)$ is the set of the subjective preferences functions in the canonical view (4) got upon the condition of (3) from the postulated functional (1) in the general case in the view of the variant of (2).

In all ten particular previous cases the differential equations can be obtained from (6) with corresponding $\pi_i(t)$ and α_i, also (4, 7):

$$\ddot{x}-\frac{\dot{x}}{\alpha}-\frac{1+e^{\beta(x-\alpha\dot{x})}}{\alpha^2\beta}=0,\ \ddot{x}+\frac{\dot{x}(\alpha\dot{x}-1)}{\alpha x}-\frac{e^{\beta x\cdot(1-\alpha\dot{x})}+1}{\alpha^2\beta x^2}=0,$$

$$\ddot{x}=\frac{x^2\left(e^{\beta\left(x-\alpha\frac{\dot{x}}{x}\right)}+1\right)}{\alpha^2\beta}+\frac{(\alpha\dot{x}+x^2)\dot{x}}{\alpha x},$$

$$x=\frac{1}{\alpha},\ x=\alpha,\ x=\sqrt{\alpha},$$

$$\ddot{x}=\frac{\pi_1+\alpha_2\{\beta\pi_2(\pi_1+\alpha_3\dot{x}\pi_3)\dot{x}\}-\alpha_3 x\{\beta\pi_3[\alpha_3\dot{x}(\pi_1+\pi_2)-\pi_1]\dot{x}\}}{\alpha_2\{\beta\pi_2[\alpha_2(\pi_1+\pi_3)-\alpha_3 x\pi_3]\}+\alpha_3 x\{\beta\pi_3[\alpha_3 x(\pi_1+\pi_2)-\alpha_2\pi_2]\}},$$

$$\ddot{x}=\frac{\pi_1+\alpha_2\left\{\beta\pi_2\left(\pi_1-\alpha_3\frac{\dot{x}}{x^2}\pi_3\right)\dot{x}\right\}-\frac{\alpha_3}{x}\left\{\beta\pi_3\left[-\alpha_3\frac{\dot{x}}{x^2}(\pi_1+\pi_2)-\pi_1\right]\dot{x}\right\}}{\alpha_2\left\{\beta\pi_2\left[\alpha_2(\pi_1+\pi_3)-\frac{\alpha_3}{x}\pi_3\right]\right\}+\frac{\alpha_3}{x}\left\{\beta\pi_3\left[\frac{\alpha_3}{x}(\pi_1+\pi_2)-\alpha_2\pi_2\right]\right\}},$$

$$\ddot{x}=\frac{\pi_1-\alpha_2 x\left\{\beta\pi_2\left[\alpha_2\dot{x}(\pi_1+\pi_3)-\pi_1+\alpha_3\frac{\dot{x}}{x^2}\pi_3\right]\dot{x}\right\}}{\alpha_2 x\left\{\beta\pi_2\left[\alpha_2 x(\pi_1+\pi_3)-\frac{\alpha_3}{x}\pi_3\right]\right\}+\frac{\alpha_3}{x}\left\{\beta\pi_3\left[\frac{\alpha_3}{x}(\pi_1+\pi_2)-\alpha_2 x\pi_2\right]\right\}}-$$

$$-\frac{\frac{\alpha_3}{x}\left\{\beta\pi_3\left[-\alpha_3\frac{\dot{x}}{x^2}(\pi_1+\pi_2)-\pi_1-\alpha_2\dot{x}\pi_2\right]\dot{x}\right\}}{\alpha_2 x\left\{\beta\pi_2\left[\alpha_2 x(\pi_1+\pi_3)-\frac{\alpha_3}{x}\pi_3\right]\right\}+\frac{\alpha_3}{x}\left\{\beta\pi_3\left[\frac{\alpha_3}{x}(\pi_1+\pi_2)-\alpha_2 x\pi_2\right]\right\}},$$

$$\ddot{x}=\frac{A+B+C}{D+E+F}. \tag{8}$$

where

$$A=\beta\dot{x}\pi_1\left(\alpha_2\pi_2-\frac{\alpha_3}{x^2}\pi_3\right)\dot{x},\ B=-\alpha_2 x\left\{\beta\dot{x}\pi_2\left[\alpha_2(\pi_1+\pi_3)+\frac{\alpha_3}{x^2}\pi_3\right]\dot{x}\right\},$$

$$C=-\frac{\alpha_3}{x}\left\{\beta\dot{x}\pi_3\left[-\frac{\alpha_3}{x^2}(\pi_1+\pi_2)-\alpha_2\pi_2\right]\dot{x}\right\},\ D=\beta\pi_1\left[\pi_2(1-\alpha_2 x)+\pi_3\left(1-\frac{\alpha_3}{x}\right)\right],$$

$$E=\alpha_2 x\left\{\beta\pi_2\left[\alpha_2 x(\pi_1+\pi_3)-\pi_1-\frac{\alpha_3}{x}\pi_3\right]\right\},$$

$$F=\frac{\alpha_3}{x}\left\{\beta\pi_3\left[\frac{\alpha_3}{x}(\pi_1+\pi_2)-\pi_1-\alpha_2 x\pi_2\right]\right\}, \tag{9}$$

the preferences are

$$\pi_1=\frac{e^{\beta\dot{x}}}{e^{\beta\dot{x}}+e^{\alpha_2\beta x\dot{x}}+e^{\alpha_3\beta\frac{\dot{x}}{x}}},\ \pi_2=\frac{e^{\alpha_2\beta x\dot{x}}}{e^{\beta\dot{x}}+e^{\alpha_2\beta x\dot{x}}+e^{\alpha_3\beta\frac{\dot{x}}{x}}},\ \pi_3=\frac{e^{\alpha_3\beta\frac{\dot{x}}{x}}}{e^{\beta\dot{x}}+e^{\alpha_2\beta x\dot{x}}+e^{\alpha_3\beta\frac{\dot{x}}{x}}}. \tag{10}$$

Conclusions. As it is seen from the (8), the extremals of $x=\frac{1}{\alpha}$, $x=\alpha$, $x=\sqrt{\alpha}$ have the property of invariance. These extremals are of the functionals with the cognitive functions of the view

$$\beta[\pi_1(t)\dot{x}(t)+\alpha\pi_2(t)x(t)\dot{x}(t)],\ \beta\left[\pi_1(t)\dot{x}(t)+\alpha\pi_2(t)\frac{\dot{x}(t)}{x(t)}\right],$$

$$\beta\left[\pi_1(t)x(t)\dot{x}(t)+\alpha\pi_2(t)\frac{\dot{x}(t)}{x(t)}\right], \tag{11}$$

correspondingly.

All the particular functionals can be obtained from (2), preferences from (4), equations by Euler-Lagrange in the transformed view from (5), differential equations from (6). Thus, we have got the needed theoretical result, through the method of (1-11). The same results can be obtained in different ways, for example, from a variant of the functionals set (2) or from generalized form of (4-6).

Prospects of further researches. The next step of the researches is to find solutions of the differential equations (6, 8), i.e. the extremals of the functionals (2). For further researches it is expedient to investigate theoretically variational problems of subjective analysis with movable boundaries, corner points, one side variations, on conditional extremum. Also, it is a kind of a scientific interest to investigate the subjective entropy of the individual preferences, analyze variational principle with the functional hybrid models that involves the threshold entropies of an active system preferences.

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Касьянов В.О., Гончаренко А.В. ВАРІАЦІЙНИЙ ПРИНЦИП У ПРОБЛЕМІ ЕКСПЛУАТАЦІЇ СУДНОВОЇ ЕНЕРГЕТИЧНОЇ УСТАНОВКИ З УРАХУВАННЯМ СУБ'ЄКТИВНИХ ПЕРЕВАГ

Розроблено теоретичну основу для найпростішого випадку варіаційної задачі, що враховує індивідуальні суб'єктивні переваги та їхню ентропію, експлуатаційників суднової енергетичної установки; експлуатаційну функцію та її похідну за часом. У даному випадку отримано узагальнюючі вирази функціоналу; канонічних розподілів суб'єктивних переваг; перетвореного рівняння Ейлера-Лагранжа та диференціального рівняння другого порядку.

Ключові слова: суб'єктивний аналіз, експлуатація суднової пропульсивної установки, енергетична установка, суб'єктивна ентропія, індивідуальні переваги, багатоальтернативні ситуації, рівняння Ейлера-Лагранжа, канонічні розподіли, варіаційний принцип.

Касьянов В.О., Гончаренко А.В. ВАРИАЦИОННЫЙ ПРИНЦИП В ПРОБЛЕМЕ ЭКСПЛУАТАЦИИ СУДОВОЙ ЭНЕРГЕТИЧЕСКОЙ УСТАНОВКИ С УЧЕТОМ СУБЪЕКТИВНЫХ ПРЕДПОЧТЕНИЙ

Разработана теоретическая основа для простейшего случая вариационной задачи, учитывающей индивидуальные субъективные предпочтения и их энтропию, эксплуатационников судовой энергетической установки; эксплуатационную функцию и ее производную по времени. В данном случае получены обобщающие выражения функционала; канонических распределений субъективных предпочтений; преобразованного уравнения Эйлера-Лагранжа и дифференциального уравнения второго порядка.

Ключевые слова: субъективный анализ, эксплуатация судовой пропульсивной установки, энергетическая установка, субъективная энтропия, индивидуальные предпочтения, многоальтернативные ситуации, уравнение Эйлера-Лагранжа, канонические распределения, вариационный принцип.